Light Neutralinos as Dark Matter in the Unconstrained Minimal Supersymmetric Standard Model[*]

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Abstract

The allowed parameter space for the lightest neutralino as the dark matter is explored using the Minimal Supersymmetric Standard Model as the low-energy effective theory without further theoretical constraints such as GUT. Selecting values of the parameters which are in agreement with present experimental limits and applying the additional requirement that the lightest neutralino be in a cosmologically interesting range, we give limits on the neutralino mass and composition. A similar analysis is also performed implementing the grand unification constraints. The elastic scattering cross section of the selected neutralinos on ^{27}Al and on other materials for dark matter experiments is discussed.

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1 Introduction

Particle candidates for dark matter are classified as hot or cold depending on whether they were relativistic or not at the time they decoupled from thermal equilibrium. Any particle candidate for cold dark matter implies an extension of the Standard Model. At present the most interesting candidate is the lightest neutralino of the Minimal Supersymmetric Standard Model (MSSM).

Any weakly interacting massive particle (WIMP) considered as a dark matter candidate is subject to at least two constraints: 1) its relic abundance must be cosmologically interesting, say $0.025 < \Omega h^2 < 1$, in units of the critical density and 2) its existence must be in accord with present experimental limits, provided mainly by the LEP experiments.

In the standard approach, based in solving the kinetic Boltzman equation, the relic abundance of WIMPs is given roughly by [1]:

$$\Omega h^2 \approx \frac{10^{-37} \text{cm}^2}{\langle \sigma_{ann} v \rangle} \tag{1}$$

where $\langle \sigma_{ann} v \rangle$ is the thermal average at freezeout of the annihilation cross section times the relative velocity in units of c. This is a very remarkable result: $\Omega h^2 \sim 1$ for typical weak cross sections. Suppose the annihilation of a WIMP of mass m proceeds via the exchange of a particle of mass M (where $M^2 = x M_Z^2$) coupled with the strength g ($g^2 = y g_{Z\nu\nu}^2$), where we have introduced the scaling factors x and y to express the mass M and the coupling g in terms of the Z boson mass and its coupling to neutrinos. Using Eq. (1) and the expressions for $\langle \sigma_{ann} v \rangle$ which follow from dimensional arguments we have:

$$\langle \sigma_{ann} v \rangle \ \sim \ \frac{y^2}{x^2} \, \frac{m^2}{M_Z^4} \, \frac{g_{Z\nu\nu}^4}{16\pi^2} \ = \ \frac{y^2}{x^2} \left(\frac{m}{1\,\text{GeV}} \right)^2 \times 0.4 \times 10^{-37} \, \text{cm}^2 \qquad \text{for } m \ll M \tag{2}$$

$$\langle \sigma_{ann} v \rangle \ \sim \ y^2 \, \frac{1}{m^2} \, \frac{g_{Z\nu\nu}^4}{16\pi^2} \ = \ y^2 \left(\frac{m}{1\,\text{TeV}} \right)^{-2} \times 0.2 \times 10^{-37} \, \text{cm}^2 \qquad \text{for } m \gg M$$

and requiring $\Omega h^2 < 1$ leads to a generalized Lee-Weinberg bound of:

$$\frac{x}{y} \times \mathcal{O}(1\,\text{GeV}) < m < y \times \mathcal{O}(1\,\text{TeV}) \ . \tag{3}$$

This is a useful qualitative constraint on the mass of a WIMP when used together with experimental limits on the parameters x and y (of course x and y are also constrained by the requirement that Ωh^2 should not be too small). It is convenient to distinguish two cases: a neutrino-like WIMP which interacts only via Z-boson exchange and a non neutrino-like WIMP which can annihilate via exchange of new particles.

In the first case, the presently available experimental constraint on the "invisible" width of the Z, $\Delta\Gamma_{inv}/\Gamma_\nu < 0.05$ [2], gives $y < 0.05$ (for of course x=1 and $m < M_Z/2$). Therefore, from Eq. (3) we obtain $m > \mathcal{O}(20\,\text{GeV})$. This is an order of magnitude estimate; in particular we neglect the difference between Majorana and Dirac particles.

The second case is realized in the Minimal Supersymmetric Standard Model with the lightest neutralino as the dark matter candidate. Here the analysis is much more involved because of the complexity of the model and its large parameter space. A considerable

amount of work has been devoted to the neutralino as a dark matter candidate [3]. However in most cases several additional theoretical assumptions are used (such as radiative electroweak symmetry breaking and universal boundary conditions at the GUT scale for the parameters of the soft breaking of supersymmetry) which may be too restrictive and certainly go beyond the MSSM as the low-energy effective theory.

In view of the dark matter search experiments, we explore in Sec. 2 the most general scenario for neutralinos as dark matter within the MSSM as the low-energy effective theory without any further theoretical constraints such as the grand unification constraints (GUT). For completeness, the obtained neutralino masses and compositions are compared with the results derived using the GUT constraints.

In Sec. 3 we consider the direct detection of neutralino dark matter via elastic scattering on nuclei. The dependence of the axial coupling (spin dependent) on the nuclear model and on the quark spin contents is discussed for selected neutralino compositions. In Sec. 4 we calculate the neutralino cross section on some of the nuclei presently used or planned to be used for dark matter detection. We analyze the low mass region and give the dependence of the cross section on the neutralino mass and composition. Prospects for the detection of light neutralinos are discussed in Sec. 5.

Our analysis differs from previous work on neutralino detection [3] because our more general assumptions allow the possibility of having low mass neutralinos as dark matter candidates and we evaluate their cross section on nuclei. In the present work neutralinos with relic abundance below the cosmological bound $\Omega h^2 < 0.025$ are not considered as dark matter candidates. This is different from the approach used in other work [4] where neutralinos with relic abundance below the cosmological bound are still considered as dark matter candidates and their contribution to the dark matter halo is evaluated by rescaling the local dark matter density.

2   Neutralino in the MSSM as dark matter candidate

In the present section we address the question of the most general limits on the lightest neutralino mass and its composition which follow only from the two constraints $0.025 < \Omega h^2 < 1$ and consistency with the present data from accelerator experiments, without any further theoretical assumptions. As we shall see, with this approach it is possible to obtain quite strong qualitative conclusions.

The stable neutralino is the lowest mass superposition of neutral gauginos and higgsinos:

$$\chi_1 = Z_{11}\tilde{B} + Z_{12}\tilde{W}^0 + Z_{13}\tilde{H}_1^0 + Z_{14}\tilde{H}_2^0 \tag{4}$$

or, in the photino, zino, higgsino basis:

$$\chi_1 = a\tilde{\gamma} + b\tilde{Z} + Z_{13}\tilde{H}_1^0 + Z_{14}\tilde{H}_2^0 \tag{5}$$

with $\tilde{\gamma} = \cos\theta_W \tilde{B} + \sin\theta_W \tilde{W}^0$, $\tilde{Z} = -\sin\theta_W \tilde{B} + \cos\theta_W \tilde{W}^0$ and θ_W the Weinberg angle. The neutralino composition is defined by the neutralino mass matrix, which in the basis

of Eq. (4) is:

$$\begin{pmatrix} M_1 & 0 & -M_Z \cos\beta \sin\theta_W & M_Z \sin\beta \sin\theta_W \\ 0 & M_2 & M_Z \cos\beta \cos\theta_W & -M_Z \sin\beta \cos\theta_W \\ -M_Z \cos\beta \sin\theta_W & M_Z \cos\beta \cos\theta_W & 0 & -\mu \\ M_Z \sin\beta \sin\theta_W & -M_Z \sin\beta \cos\theta_W & -\mu & 0 \end{pmatrix} \quad (6)$$

2.1 Scan of the parameter space

The neutralino mass matrix depends on several, in general free, parameters of the model: the gaugino masses M_1 and M_2, the higgsino mass parameter μ, and $\tan\beta=v_2/v_1$, where v_1 and v_2 are the vacuum expectation values of the two Higgses present in the model. All of these are independent parameters of the low-energy effective lagrangian. In the present study of the unconstrained low-energy effective theory, the values of these parameters are chosen randomly in the ranges listed in Tab. 1. The scanning procedure is such that $\log(M_2)$, $\log(\mu)$, $\log(\tan\beta)$ and the ratio M_1/M_2 are chosen with flat probability. We restrict our analysis to neutralinos lighter than $100\,\mathrm{GeV}$.

Calculations with the grand unification constraints are also performed using the same ranges but with $M_1=\frac{5}{3}M_2 \tan^2\theta_W \simeq 0.5 M_2$ and with $M_2 \simeq 0.3 m_{\tilde{g}}$. We use for the gluino mass $m_{\tilde{g}}$ the experimental lower bounds from CDF [5] and D0 [6] resulting in $M_2 \geq 50$ GeV. It should be noted that the present experimental limits on the gluino mass are derived only for a specific choice of the MSSM parameters.

We scan the parameter space incorporating the following existing experimental limits from accelerator experiments. 1) The limit on the Z boson invisible width is $\Gamma_{inv} <$ $8.4\,\mathrm{MeV}$ [2]. 2) Heavier neutralinos are unstable and could be observed in e^+e^- collisions via their decay products for the range of χ masses which are kinematically accessible. We use the mass-dependent limits on the coupling constants given in Ref. [7]. 3) The limit on the chargino mass is $M_{\chi^\pm} > 45\,\mathrm{GeV}$ [7]. The chargino mass matrix is a function of the same parameters:

$$\begin{pmatrix} M_2 & \sqrt{2}M_W \sin\beta \\ \sqrt{2}M_W \cos\beta & \mu \end{pmatrix} \quad (7)$$

and does not depend on the ratio M_1/M_2.

The scanned parameter space is shown in Fig. 1 with and without grand unification constraints after incorporating the limits from accelerator experiments. The empty regions in the $M_2 - \mu$ plane are excluded by the experimental limits. The quoted chargino mass limit is at present by far the strongest constraint on the parameter space as shown in Fig. 1 where the contour line $M_{\chi^\pm} = 45\,\mathrm{GeV}$ is plotted for $\tan\beta=1$ and $\tan\beta=50$. Without grand unification constraints, the region corresponding to small values of M_2 and $|\mu|$ is excluded for $\tan\beta > 3$.

2.2 Relic abundance

We then calculate the relic neutralino abundance in the allowed parameter region. We solve the Boltzman equation following Ref. [8], using a complete set of supersymmetric annihilation amplitudes [9]. For this we need the squark, slepton, and Higgs masses. We

assume all squarks have the same mass M_{sq} and all sleptons the same mass M_{sl}. Since the purpose of our study is to explore the parameter space, we need limits which are model-independent, whereas most experimental limits are based on certain assumptions on the values of the parameters. Thus we have chosen to repeat our calculation for several different values of the squark, slepton and CP-odd Higgs masses as shown in Tab. 2. In the case of sets A–C, the upper bound in the neutralino mass is given by the slepton mass since we require $M_\chi \leq M_{sl}$.

The lowest values of the squark masses are taken as 45 GeV, since lighter particles are ruled out by LEP data on the Z. CDF [5] has shown that the squark mass limits obtained from $p\bar{p}$ collisions are dependent on the assumed squark decay modes and that for many configurations of the MSSM parameters the squark mass is unconstrained. A recent report by D0 [6] gives limits on the squark mass as function of the gluino mass for a particular set of MSSM parameters, assuming that all squarks have the same mass. Even for this particular set of parameters, squarks can be as light as $\sim$50 GeV for $m_{\tilde{g}} \geq$300 GeV. OPAL [10] has set limits of about 45 GeV specifically for the $\tilde{t}$ mass, which may be significantly lighter than the other squarks due to the left-right mixing.

For sets A–C the CP-odd Higgs mass M_A is randomly chosen in the range 25-70 GeV in the regions of the plane M_A-M_h allowed by present LEP data [11]. In sets D and E, M_A is fixed at 200 GeV and 1 TeV respectively. The other Higgs masses are calculated from M_A and $\tan\beta$ using radiative corrections [12] which introduce a dependence on the top quark mass M_t and M_{sq}. The top quark mass is set to 170 GeV.

The result of our calculation for the relic abundance Ωh^2 obtained without the grand unification constraints are plotted in Fig. 2 for sets B–E. The cosmologically relevant neutralinos are selected imposing the condition $0.025 < \Omega h^2 < 1$. It must be noted that we do not rescale the local dark matter density for neutralinos with relic abundance below the cosmological bound $\Omega h^2 < 0.025$ since we do not consider them as dark matter candidates.

For low squark and slepton masses (set B) the condition $\Omega h^2 < 1$ is obtained for neutralino masses above 2 GeV. Decreasing the squark masses to 45 GeV (set A) gives a relic abundance very similar to the one of set B and the same value of 2 GeV for the lowest neutralinos, which can then be regarded as an absolute lower bound on the neutralino mass in the MSSM. This is one of the main conclusions of this study. The lower bound in the neutralino mass rises with M_{sq} and M_{sl} to reach $\sim$20 GeV in sets D and E.

The relic abundance evaluated using the grand unification constraints is plotted in Fig. 3 for sets B–E. Here neutralinos have a lower bound on the mass of 20 GeV, due to the experimental constraints on the gluino mass. A relic abundance $\Omega h^2 << 1$ is obtained for sets A (not shown) and B where we use low squark and slepton masses.

2.3 Neutralino composition

In Fig. 4 we show the gaugino purity $Z_{11}^2 + Z_{12}^2$ and separately the bino purity Z_{11}^2 for the cosmologically relevant neutralinos derived without grand unification constraints combining data sets A–E. The points at bino purity $Z_{11}^2 = \cos^2\theta_W \simeq 0.77$ correspond to pure photinos. It is seen that the cosmologically relevant neutralinos are dominantly gauginos (mostly bino and photino), with a substantial higgsino component coming in

above 30 GeV. The neutralino composition calculated with GUT is shown in Fig. 5 and it is similar to the composition obtained without grand unification constraints except that the photino population of the gaugino-like neutralinos is strongly reduced. The higgsino composition is shown in Fig. 6 without grand unification constraints. Since we consider values of $\tan\beta > 1$ we have $Z_{13}^2 > Z_{14}^2$ with $Z_{13} = -Z_{14}$ when $\tan\beta = 1$. The higgsino composition with grand unification constraints is similar to the one shown in Fig. 6.

The depopulation in the region with large gaugino-higgsino mixing is due to the scanning of the allowed parameter space and to the requirement of having cosmologically relevant neutralinos. The neutralino is a mixed state only when the smaller of the two gaugino masses, M_2 or M_1, has a magnitude similar to $|\mu|$. This is a fine-tuning which is not included in our scanning procedure, in consequence we have a substantial reduction in the population of mixed neutralinos. Especially for large M_2 there are few points in the parameter space corresponding to mixed neutralinos. In this case, for values of M_1 not much smaller than M_2, mixing is allowed for large values of $|\mu|$ giving neutralinos heavier than our upper bound in the neutralino mass of 100 GeV. The only mixing allowed for large M_2 corresponds to both M_1 and $|\mu|$ being small.

The dependence of the relic abundance on the neutralino composition is shown in Fig. 7 for sets B and D without grand unification constraints. Pure gauginos do not have couplings to the Z boson and have higher relic abundance than pure higgsinos. The relic abundance of pure photinos is bigger than that of pure bino because the former does not couple to the Higgs bosons. At large $\tan\beta$ the annihilation is dominated by Higgs exchange which is proportional to the gaugino-higgsino mixing and goes to zero for pure states. As a result, large gaugino-higgsino mixtures have low relic abundance and are excluded by the request of having $\Omega h^2 > 0.025$. The dependence of the cosmologically relevant neutralino compositions on $\tan\beta$ is shown in Fig. 8 for sets B and D. Gaugino-higgsino mixtures are relevant only at small $\tan\beta$ while at large $\tan\beta$ our cosmologically relevant neutralinos are mainly pure gauginos with exception of set E which has a very heavy CP-odd Higgs boson. There is a substantial photino component for $\tan\beta < 2$ while at large $\tan\beta$ the photino component is strongly reduced due to the structure of the neutralino mass matrix. Due to the large squark mass, photino have relic abundance $\Omega h^2 > 1$ in the case of sets D and E.

3 Neutralino detection

Dark matter neutralinos can be directly detected via their elastic scattering on nuclei. The scattering cross section has two components. An effective axial-vector interaction gives a spin-dependent (SD) cross section which is non-zero only for nuclei with net spin. Scalar and vector interactions give spin-independent (SI) cross sections which involve the squares of nuclear neutron and proton numbers. The relative strengths of the two parts of the neutralino interaction on nuclei depends on the neutralino composition. A pure gaugino couples only to squarks and sleptons and has only a SD interaction. The sleptons are not relevant for interactions with nuclei. A pure higgsino couples only to the axial-vector part of the Z boson and thus has only a SD interaction. (The Yukawa coupling to squarks is negligible for scattering on the proton.) Mixed states have both SD and SI

interactions, the later is due to squark exchange and Higgs exchange and is proportional to the zino-higgsino mixture.

The actual scattering cross section depends on the coefficients Z_{1j} that specify the neutralino composition, and on the masses M_{sq} and M_A of the exchanged particles. To obtain the neutralino scattering on nuclei for each set of supersymmetric parameters, we first calculate the coefficients of the effective 4-fermion operators describing neutralino-quark scattering. These operators are used to give the neutralino-nucleon amplitudes, which are then related to the nuclear matrix elements.

We shall focus on calculating the cross section for neutralino scattering on a nucleus at zero momentum transfer. This incorporates the essentials of the physics of neutralino cross section on nuclei. In Sec. 5 we will include in the evaluation of the recoil energy spectrum and detection rate nuclear form factor effects due to the momentum transfer. In the following, we will call the integral of the zero momentum transfer cross section over all momentum transfer σ_0. The cross section σ_0 is derived from:

$$\sigma_0 = 4\frac{G_F^2}{\pi}\Big[S_A(0) + S_S(0)\Big]M_{red}^2 \tag{8}$$

where the effective axial-vector current $S_A(0)$ and the scalar current $S_S(0)$ coefficients are calculated at zero momentum transfer (q=0) and M_{red} is the reduced mass. The coefficients S_A and S_S are evaluated using the expressions derived in Ref. [13].

3.1   Spin Dependent cross section

The evaluation of the SD cross section is problematic because it depends on the model used to describe the spin structure function of the nucleon and on the nuclear model used to derive the total proton and neutron spins. In this section we discuss the resulting uncertainty in the determination of the axial-vector current coefficient:

$$S_A(0) = 8J(J+1)\Big\{\frac{a_p < S_p >}{J} + \frac{a_n < S_n >}{J}\Big\}^2 \tag{9}$$

with:

$$a_{p(n)} = \Big[\sum_{u,d,s} A_q \Delta Q_q\Big]_{p(n)} \tag{10}$$

where J is the nuclear spin and $< S_{p(n)} >$ are the total proton and neutron spins. The terms A_q refer to the coupling of the up, down and strange quarks weighted with the nucleon quark spin content coefficients ΔQ_q. The entire dependence on the nature of the neutralino, and therefore of our cosmological considerations, enters only through the coefficients A_q. For the determination of $< S_{p(n)} >$ and ΔQ_q, we rely entirely on previous work [14]-[20].

It is illustrative to consider pure photino and pure bino interactions for which the proton axial coupling coefficients a_p are [13]:

$$a_p = -\frac{M_W^2}{M_{sq}^2}\tan^2\theta_W\Big\{\frac{17}{36}\Delta Q_u + \frac{5}{36}(\Delta Q_d + \Delta Q_s)\Big\} \qquad \text{pure bino} \tag{11}$$

$$a_p = -2\frac{M_W^2}{M_{sq}^2}\sin^2\theta_W\Big\{\frac{4}{9}\Delta Q_u + \frac{1}{9}(\Delta Q_d + \Delta Q_s)\Big\} \qquad \text{pure photino}$$

Since the quark spin coefficients ΔQ_q are evaluated for scattering on a proton, the neutron axial coupling coefficient a_n is obtained by interchanging the values of ΔQ_u and ΔQ_d according to the different quark structure of the two nucleons.

In Tab. 3 we have calculated $a_{p(n)}$ from Eq. (11) using for ΔQ_q the values derived from the Naive Quark Model (NQM) [14], the European Muon Collaboration (EMC) measurements [15] and from two analyses of the present data on polarized lepton-nucleon scattering Global Fit-1 from Ref. [16] and the more recent Global Fit-2 from Ref. [17]. The axial coupling coefficients a_p are roughly a factor of two larger than a_n due to the different quark structure of the two nucleons which enhances the axial coupling on protons and therefore on proton-odd nuclei.

The dependence of the axial-vector current coefficient $S_A(0)$ on the model used to describe the nuclear structure is shown in Tab. 4 where we have calculated the axial-vector current coefficient for pure photino and pure bino interactions on the proton-odd nucleus ^{27}Al and on the neutron-odd nucleus ^{73}Ge. We compare the predictions from the Odd Group Model (OGM) [18] and the detailed Shell Model reported in Ref. [19] for ^{27}Al and in Ref. [20] for ^{73}Ge. We also show the effects of different model for the spin quark coefficients ΔQ_q.

If the OGM is used to describe the nuclear structure, only the unpaired nucleon contributes to the axial coupling. In this case the axial-vector current coefficients $S_A(0)$ are proportional to a_p^2 for proton-odd nuclei and a_n^2 for neutron-odd nuclei. In the OGM, where only the valence nucleon plays a role, the cross sections on nuclei obtained with the NQM are roughly a factor of two larger on proton-odd nuclei than those for other quark models directly reflecting the cross sections on single nucleons. On the contrary, the SD cross section on neutron-odd nuclei derived with the NQM is more than an order of magnitude smaller than the one obtained using the other ΔQ_q values due to the small value of a_n in the NQM.

It should be noted that the values for the total proton and neutron spins derived in the detailed Shell Model calculations for ^{27}Al have an uncertainty of roughly 30% related to the quenching of the spin matrix elements. Quenching is more important in ^{73}Ge and we use the quenching factor $Q = 0.833$ in the isovector piece of the axial coupling coefficients $a_{p(n)}$ as suggested in Ref. [20]. In this case the axial coupling coefficients have to be replaced with $a_p' = 0.917 a_p + 0.0835 a_n$ and $a_n' = 0.0835 a_p + 0.917 a_n$ where $a_{p(n)}$ are evaluated in the unquenched case (Eq. (11)). The cross section on ^{27}Al evaluated using the detailed Shell Model calculations are roughly 2 times larger than the results obtained with the OGM for all the four model used to describe the spin structure of the nucleon. The detailed Shell Model gives almost 4 times larger SD cross sections on ^{73}Ge than the OGM for all the three quark models with non zero strange quark content. The axial current coefficients for pure photinos and binos are smaller for interactions on ^{73}Ge than on ^{27}Al mainly because of the different quark structure of neutrons and protons.

In general neutralinos are mixtures of gauginos and higgsinos. For pure higgsinos, the axial coupling coefficient a_p derived from Ref. [13] is:

$$
\begin{aligned}
a_p &= \frac{1}{4}\left(Z_{13}^2 - Z_{14}^2\right)\left[\Delta Q_u - \Delta Q_d - \Delta Q_s\right] \\
&\quad -\frac{1}{2}\frac{1}{\sin^2\beta}\frac{1}{M_{sq}^2}\left\{Z_{14}^2 M_u^2 \Delta Q_u + Z_{13}^2 \tan^2\beta\left[M_d^2 \Delta Q_d + M_s^2 \Delta Q_s\right]\right\}
\end{aligned}
\tag{12}
$$

where $M_{u,d,s}$ are the quark masses. The neutron axial coupling coefficient a_n is obtained by interchanging the values of ΔQ_u and ΔQ_d. For the same higgsino composition, the difference between a_p and a_n is mainly due to the first term of Eq. (12) which is always positive for protons and negative for neutrons since $Z_{13}^2 \geq Z_{14}^2$ (see Fig. 6). The second term of Eq. (12) does not change significantly going from a_p to a_n because it is dominated by the contribution of the strange quark which is much heavier than the u and d quarks. As a result, $|a_p| > |a_n|$ and higgsino interactions have the largest cross section on neutron-odd nuclei.

3.2 Spin Independent cross section

We evaluate the scalar current coefficient S_S from:

$$S_S(0) = 2A^2[P_s\lambda_s + \sum_{c,b,t} P_h\lambda_h]^2 \qquad (13)$$

where A is the atomic number. The terms P_i are determined by the couplings in the effective $\chi\chi q\tilde{q}$ lagrangian, with contributions from squark exchange and Higgs exchange of the lightest and the heaviest Higgs bosons. The supersymmetric content of the interaction is represented by the P_i terms. The coefficients λ_i are given by the matrix elements of the quark operators $\frac{M_i}{M_W}q_i\tilde{q}_i$ taken between the nucleon states and are by standard methods [3] expressed in terms of the sigma-term measured in pion-nucleon interactions and the nucleon mass. For λ_i and P_i[1] we use the values given in Ref. [13].

The contribution of the light u and d quarks to the scalar neutralino-nucleon coupling can be neglected and the term $P_i\lambda_i$ accounts for the coupling to the strange quark (index s) and to the heavy quarks c, b and t (index h). Since we do not consider the contribution of the u and d quarks, we do not have to distinguish between scalar cross section on neutron or proton.

It must be noted that the SI cross section evaluated using the $P_i\lambda_i$ terms of Ref. [13] is four times smaller than the SI cross section derived in Ref. [4]. This factor of four is independent of the neutralino composition and mass.

4 Cross section on nuclei

In this section we derive the SD and SI cross sections on some of the material presently used or planned to be used for dark matter detection. In contrast to our illustrative examples in the previous section, we will now consider arbitrary neutralino compositions, as given by our scan for cosmologically relevant cases.

4.1 ^{27}Al

We start with the proton-odd nucleus ^{27}Al (100% natural abundance). Dark matter detectors made of sapphire (Al_2O_3) are presently under preparation by the CRESST [21] and EDELWEISS collaborations [22].

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[1]The original notation of Ref. [13] is S_q instead of P_i

In order to evaluate the SD cross section we chose to use the quark spin coefficients ΔQ_q derived in Ref. [17] (Global Fit-2) and the results of the detailed Shell Model calculations reported in Ref. [19] for the nuclear physics. In Fig. 9 the SI cross section is plotted versus the SD part for the cosmologically relevant neutralinos without the grand unification constraints. There is a 5 orders of magnitude spread in the values of the SD cross section for the neutralino masses and compositions considered in our analysis. The spread is even more pronounced in the case of the SI cross section.

The dependence of the SI cross section on the neutralino composition is shown in Fig. 10 for data sets B and D and $\tan\beta$ <2. Since the scalar coupling is proportional to the zino-higgsino mixture, pure neutralino states have zero SI cross section. Mixtures with a large photino component have very small SI cross section $< 10^{-6}$ which is almost 5 orders of magnitude smaller than the corresponding SD part. The cross section is reduced to values below 10^{-3} pbarn for binos and higgsinos with purity > 0.99.

The dependence of the SI cross section on $\tan\beta$ is shown in Fig. 11. Although the SI cross section is expected to increase with increasing $\tan\beta$, the maximum value of the SI cross section for our selected neutralinos changes only slightly with $\tan\beta$. This is related to the dependence of the neutralino composition on $\tan\beta$ as discussed in Sec. 2.3. At small $\tan\beta$ gaugino-higgsino mixtures are cosmologically relevant giving the maximum SI cross section. At large $\tan\beta$ our selected neutralinos are mainly bino or higgsino with a reduced SI cross section.

The SD and SI cross sections obtained without the grand unification constraints are plotted versus the neutralino mass in Fig. 12 for the combined data sets A–E. The bands of points in the low mass region of the SD cross section are due to pure photinos and pure binos as shown in Fig. 13. The SD cross section of pure binos is ∼2 times smaller than the one for pure photinos as shown in Tab. 4. The photino contribution to the SD cross section is relevant only for small $\tan\beta$ because the photino population in our selected neutralinos is strongly reduced at high $\tan\beta$. The highest SD cross section is obtained for the pure gauginos of set A. In this case squark exchange gives a significant contribution because of the small squark mass M_{sq}=45 GeV.

In Fig. 14 the SD cross section is plotted versus the bino purity for sets B and D without grand unification constraints. For heavy squarks (set D) the highest SD cross section is obtained for gaugino-higgsino mixtures with a substantial higgsino component. This is because squark exchange is suppressed and Z exchange becomes dominant.

The SD cross section for our neutralinos is sometimes even larger than the cross section for a massive Majorana neutrino interacting via Z exchange; this happens when the squark mass is low and its exchange gives a significant contribution to the cross section. In contrast, the maximum values of the SI cross section are two orders of magnitude below that for Dirac neutrinos.

The SI and SD cross sections evaluated with the grand unification constraints are plotted in Fig. 15 for the cosmologically relevant neutralinos of the combined data sets A–E. The main contribution to the cross section is given by sets C–E since for sets A and B the neutralino relic abundance is $\Omega h^2 << 1$ (see Fig. 3). The horizontal bands in the SD cross section are due to the bino contributions of sets C and D since in the case of GUT the photino population is suppressed as shown in Fig. 5. Comparing the cross sections obtained with and without grand unification constraints, we see that the

maximal value of σ_0 and the dependence of the cross section on the neutralino mass are almost the same. In the case of GUT, the cutoff in the neutralino mass is at $\simeq 20\,\text{GeV}$ as discussed in Sec. 2.2.

4.2 ^{23}Na and ^{127}I

We turn now to the evaluation of the neutralino cross section on the proton-odd nuclei ^{23}Na and ^{127}I both with 100% natural abundance. NaI(Tl) scintillator detectors are presently used for dark matter searches by several groups [23].

The cross sections σ_0 (SI+SD) on ^{23}Na and on ^{127}I are plotted in Fig. 16 for the combined sets A–E without grand unification constraints. Since ^{27}Al and ^{23}Na have similar atomic numbers the SI cross sections in the two materials do not differ significantly. Due to the difference in the atomic numbers, the SI cross section on ^{127}I is roughly two orders of magnitude larger than on ^{23}Na.

The dependence of the SD cross section on the neutralino composition is the same for Na, I and Al because these nuclei have an unpaired proton. The dependence of the SD cross section on the absorber material is given by the reduced mass, the nuclear spin and the total proton and neutron spins $< S_{p(n)} >$. In the case of ^{23}Na and ^{127}I detailed Shell Model calculations are not available and we use the Odd Group Model [18] where only the unpaired proton contributes. The resulting axial-vector current coefficients S_A for pure bino and pure photino (purity=1) obtained using the quark spin coefficients Global Fit-2 are listed in Tab. 5. The SD cross section on ^{23}Na is almost three times smaller than the values derived for ^{27}Al with the OGM and the spin quark coefficients Global Fit-2 (see Tab. 4). This is due to the different total proton spins and nuclear spins. In the case of ^{127}I, the SD cross section evaluated with the Global Fit-2 is almost two orders of magnitude weaker than the SI part and can be neglected in the evaluation of σ_0.

The bands of points in the total cross section of Fig. 16 are due to the axial coupling of neutralinos with a large photino components, for which the SI cross section is many order of magnitude weaker than the SD part.

4.3 ^{73}Ge and ^{117}Sn

We will discuss now the case of neutron-odd nuclei starting with ^{73}Ge (7.8% natural abundance). Although the Ge dark matter detectors presently used [24] are made of natural germanium which is mainly ^{74}Ge (36.5%), ^{72}Ge (27.4%) and ^{70}Ge (20.5%), the use of enriched ^{73}Ge detectors is planned for the future [25].

In order to evaluate the SD cross section we use the quark spin coefficients ΔQ_q derived in Ref. [17] (Global Fit-2) and the results of the detailed Shell Model calculations reported in Ref. [20] for the nuclear physics. In Fig. 17 the SD cross section is plotted versus the bino purity for data sets B and D without grand unification constraints. In both cases the highest SD cross section is obtained for pure higgsinos or for mixtures with a substantial higgsino component. In the case of set B this is different from what we calculated for Al (see Fig. 14) where the maximum SD cross section was obtained for gaugino-like neutralinos. This is due to the different values of the axial coupling coefficient of pure higgsino in the case of proton or neutron odd nuclei as discussed in Sec. 3.1.

Since the contribution of the light u and d quarks is neglected in the computation of the SI cross section as discussed in Sec. 3.2, the dependence of the axial coupling on the neutralino composition does not differ from the one discussed for the proton-odd ^{27}Al nucleus. In Fig. 18 the SI cross section is plotted versus the SD part for data sets B and D without grand unification constraints. The SI cross section is generally bigger than the SD part because the quite large value of the atomic number enhances the scalar coupling. For a large selection of neutralino compositions, the SD part can then be neglected in the evaluation of the total cross section.

In Fig. 19 the SD and SI cross sections are plotted versus the neutralino mass for the combined sets A–E without the grand unification constraints. The bands of points in the low mass region of the SD cross section correspond to the contribution of pure photinos of sets A–C with set A giving the highest contribution. The structure due to pure bino is less evident than in the case of Al. It must be noted that the photino population decreases with increasing values of $\tan\beta$ as discussed in Sec. 2.3. In Fig. 20 we plot the SD and SI cross sections for the combined sets A–E with grand unification constraints. The dependence of σ_0 on the implementation of the grand unification constraints is the same as discussed for Al in Sec. 4.1.

Another material planned for dark matter detectors is Sn [26]. Natural tin consists of different isotopes among which the only odd-isotope with a relevant natural abundance is ^{117}Sn (7.7%). The axial-vector current coefficients for pure bino and pure photino interactions on the neutron-odd isotope ^{117}Sn are listed in Tab. 5. The axial coupling of pure binos and pure photinos evaluated with the OGM [18] and the Global Fit-2 is roughly three times larger on ^{117}Sn than on ^{73}Ge mainly because of the different nuclear spin J. The SI part can be derived from the SI cross section evaluated for ^{27}Al with appropriated scaling due to the different nuclear mass. The cross section σ_0 (SI+SD) on ^{117}Sn is plotted in Fig. 21 for the combined sets A–E without grand unification constraints. Due to the large value of the atomic number, the SI cross section is generally bigger than the SD part and, for a large selection of neutralino compositions, the axial coupling can be neglected in the evaluation of σ_0.

5   Prospects for detection of low mass neutralinos

The present experimental limits for WIMP dark matter are given by Ge [24] and Si [27] ionization detectors and NaI scintillators [23]. Due to their relatively high recoil-energy threshold, such detectors are not sensitive to low mass ($M_\chi < 10$ GeV) dark matter particles.

In order to explore the low mass window, a dark matter search experiment (CRESST) based on the use of a sapphire (Al_2O_3) cryogenic detector with a low energy threshold is currently under preparation [21]. The first stage of this experiment (CRESST-1) will use a 1 kg sapphire detector with an expected energy threshold of 0.5 keV and a full width half maximum energy resolution ΔE=200 eV.

We perform a rough estimate of the CRESST-1 sensitivity to neutralino dark matter assuming a flat distribution of the radioactive background at 1 count/keV/kg/day. A discussion of different radioactive background sources can be found in Ref. [21]. For

each dark matter particle mass, the simulated background data is fitted with the sum of a flat component for the background and the calculated recoil energy spectrum for ^{27}Al. The exclusion limit is defined as the cross section for which the fit gives a $\mathcal{X}^2$ value corresponding to the 90% confidence level. The expected recoil energy spectrum is calculated using an exponential form factor to account for the loss of nuclear coherence at high momentum transfer [18]. We use the same exponential form factor for both SD and SI interactions. Due to the small nuclear radius the effect of the nuclear form factor on the shape of the recoil energy spectrum is negligible for Al. We use a Maxwell-Boltzman velocity distribution for the dark matter halo with an average velocity of 270 km/s and an upper cutoff at the escape velocity of 575 km/s in the rest frame of our galaxy. The local halo density depends on the model used to describe the structure of our galaxy with a resulting uncertainty in the detection rate of roughly a factor of two [3]. In this work the density of dark matter particles is assumed to be 0.3 GeV/cm^3.

In Fig. 22 the cross section σ_0 (SI + SD), calculated without the grand unification constraints for the combined data sets A–E, is compared to the rough estimate of the expected sensitivity of the first stage of the CRESST experiment assuming a measurement time of 1 year. The expected low energy threshold of the cryogenic detector used in the CRESST experiment combined with the low atomic number of Al provide an appreciable sensitivity to dark matter particles starting at $\sim$1 GeV. It is important to note that ^{16}O does not contribute to the SD cross section on sapphire because of the even number of nucleons. Due to the small nuclear mass, the SI cross section on ^{16}O is roughly an order of magnitude lower than on ^{27}Al and can be neglected in the evaluation of the sensitivity of a sapphire detector.

For completeness we have plotted in Fig. 23 the normalized interaction rate in units of counts/kg/day/pbarn that is, we show the interaction rate assuming a cross section of 1 pbarn. The measured rate in a given detector will be less than the interaction rate, due to threshold and efficiency effects. The interaction rate is evaluated using the analytical expression derived in Ref. [28] with a zero energy threshold[2]. The variation with mass reflects the change in the dark matter flux combined with the effect of the nuclear form factor which is relevant for scattering of heavy neutralinos on large nuclei. The variation with nucleus reflects the difference in the number of target nuclei per unit weight. In the case of ^{73}Ge and ^{117}Sn the interaction rate is calculated for a 100% enriched detector. We have separated the contribution of ^{23}Na and ^{127}I to the total interaction rate of NaI detectors (50% Na and 50% I atoms). In the case of sapphire detectors we have considered only the rate due to neutralino interactions on the ^{27}Al content of sapphire since the neutralino cross section on ^{16}O is negligible.

A final question concerns how the counting rate for a given neutralino will vary when we change the target material. This is of interest in connection with two points. One is the verification of a presumed dark matter signal and the other is discovering the nature of the neutralino. Should there be an indication of a dark matter signal in a detector it will be of course of the utmost importance to verify that it is a true signal and not the result of noise or backgrounds of some kind. An aid in doing this will be variation of the target material, leading to changes which are not necessarily those of the noise or

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[2]We use the expression given for $(Rate_c)$

background. Secondly, varying the type of nucleus can help to determine the character of the particle, to see if its interactions are dominantly SI or SD, if it is stronger on neutrons or protons and so forth, thus helping to pin down the type of neutralino. Disentangling and understanding these various effects will be a question for much further detailed study. However, we can give a first impression on the basis of the calculations in this paper by presenting examples with some of the various candidate neutralinos that we have found in our study.

In Fig. 24 we show the dependence of the cross section σ_0 for different neutralino compositions on various materials in the case of set B with $\tan\beta=2$, $M_A=50\,\text{GeV}$ and the Higgs mixing angle $\alpha = -1.24$. We use Global Fit-2 for the quark spin coefficients and the OGM for the total proton and neutron spins for Na, I and Sn. For Al and Ge we use the shell model as discussed in Sec. 3.1. The highest cross section is given by zino-higgsino mixtures since pure higgsino and gaugino states do not have SI interactions, which increase with A^2. For other examples, which we take to be pure higgsino or gaugino, we show no entries for ^{74}Ge since it has spin zero and thus there is no SD interaction. For pure higgsinos we have higher cross sections on neutron-odd nuclei, as discussed in Sec. 3.1. The cross section for pure higgsinos and pure photinos on proton-odd nuclei tend to have roughly the same value. In the case of pure higgsino or gaugino states, σ_0 does not scale with A^2 since the SI part is absent and the dependence on the materials is due to the nuclear model. It should be stressed that Fig. 24 represents only a particular case. As discussed in the previous sections, the cross section on nuclei depends not only on the neutralino composition but also on the squark and Higgs masses and $\tan\beta$.

6 Summary

The minimal supersymmetric standard model as the low-energy effective theory has been used to explore the most general scenario for neutralino dark matter with the present accelerator data and the requirement $0.025 < \Omega h^2 < 1$ as the only constraints without further theoretical assumptions. In order to obtain limits on the neutralino mass which are model-independent, we scanned the parameter space randomly choosing the values of the free parameters of the model using different sets for the squark, slepton and CP-odd Higgs masses.

We have found an absolute lower bound for the mass of the cosmologically relevant neutralinos of $M_\chi > 2\,\text{GeV}$. The cosmologically relevant neutralinos are dominantly gauginos with an higgsino component coming in for masses above $30\,\text{GeV}$. We performed a similar analysis including the grand unification constraints and obtaining dark matter neutralinos with a lower bound on the mass of $20\,\text{GeV}$.

For the cosmologically relevant neutralinos, we have calculated the spin dependent (SD) and spin independent (SI) cross sections on different nuclei under consideration for dark matter detection. We have found a 5 orders of magnitude spread in the values of the SD cross section for the neutralino masses and composition considered in our analysis. The spread is even more pronounced in the case of the SI cross section.

We have discussed the dependence of the SD cross section on the nuclear model and on the nucleon quark spin content. We have used the Odd Group Model and the detailed

Shell Model calculations to evaluate the total proton and neutron spins and have found that the uncertainty resulting from the choice of the nuclear model is within a factor of two for the quark spin models with a non zero strange quark contribution. The naive Quark Model, where only up and down quarks are considered, gives the smallest cross section on neutron-odd nuclei and the highest axial coupling for proton-odd nuclei.

For our selected neutralinos, the contribution of the photino component to the SD cross section is relevant only for small $\tan\beta$ because the photino population is strongly reduced at large $\tan\beta$. The highest SD cross section is obtained for the pure photinos and pure binos of set A for which squark exchange gives a significant contribution because of the small squark mass M_{sq}=45 GeV. For heavy squarks the highest SD cross section is obtained for gaugino-higgsino mixtures with a substantial higgsino component because squark exchange is suppressed and Z exchange becomes dominant. The higgsino contribution to the SD cross section is more relevant in neutron-odd nuclei.

In the case of ^{27}Al, the SD and SI cross sections are comparable in magnitude for a large selection of neutralino compositions. The SI cross section on ^{73}Ge is generally bigger than the SD part because the larger value of the nuclear mass enhances the scalar coupling. For a large selection of the parameter space, the SD part can be neglected in the evaluation of the cross section σ_0. This is true also for ^{127}I where the SD cross section is almost two orders of magnitude smaller than the SI part and can be neglected. Due to the different values of the total proton spins and nuclear spins, the SD cross section on ^{23}Na is almost three times smaller than on ^{27}Al.

The SD cross section increases with the increasing higgsino component due to the Z exchange contributions. SI cross sections are large for neutralinos which are zino-higgsino mixtures. Large cross sections on nuclei correspond to small relic abundance, close to our lower bound $\Omega h^2 > 0.025$.

The neutralino cross section on ^{27}Al was compared with a rough estimate of the expected sensitivity of the first stage of the CRESST which uses a low energy threshold sapphire cryogenic detector. We have shown that the expected sensitivity to low mass dark matter particles of the CRESST experiment can be useful in probing the light neutralino scenario ($M_\chi > 2$ GeV) predicted by the present analysis based on the unconstrained minimal supersymmetric standard model.

Acknowledgement

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Figure captions

Fig. 1: Scanned parameter space a) without and b) with grand unification constraints. The existing limits from accelerator experiments are incorporated. The contour line corresponds to the lower limit on the chargino mass $M_{\chi^\pm} = 45\,\mathrm{GeV}$ evaluated for $\tan\beta{=}1$ (solid lines) and $\tan\beta{=}50$ (dotted lines). The horizontal line at $M_2{=}50\,\mathrm{GeV}$ in the GUT case corresponds to the experimental bound from the gluino mass. The upper left and right corners in the GUT case are empty because in these regions $M_\chi \geq 100\,\mathrm{GeV}$.

Fig. 2: Relic abundance versus neutralino mass in the parameter region allowed by the experimental constraints for the four sets B–E listed in Tab. 2. The plot for set A is very similar to that for B. The cosmologically interesting relic abundance $0.025 < \Omega h^2 < 1$ is within the two horizontal lines. In the case of sets A–C, the upper bound in the neutralino mass is given by the slepton mass since we require $M_\chi \leq M_{sl}$.

Fig. 3: As in Fig. 2 with the grand unification constraints.

Fig. 4: Gaugino purity $Z_{11}^2 + Z_{12}^2$ a) and bino purity Z_{11}^2 b) for neutralinos in the cosmologically interesting region. The data sets A–E are combined. The points at bino purity $Z_{11}^2 = \cos^2\theta_W = 0.77$ correspond to pure photinos.

Fig. 5: As in Fig. 4 with the grand unification constraints.

Fig. 6: Higgsino composition for the cosmologically interesting neutralinos without grand unification constraints. The data sets A–E are combined. The points with $Z_{13}^2 + Z_{14}^2 = 1$ correspond to pure higgsinos.

Fig. 7: Relic abundance versus bino purity Z_{11}^2 for data sets B and D without grand unification constraints. The cosmologically interesting relic abundance $0.025 < \Omega h^2 < 1$ is within the two horizontal lines.

Fig. 8: Bino purity Z_{11}^2 versus $\tan\beta$ for the cosmologically relevant neutralinos of data sets B and D without grand unification constraints.

Fig. 9: SI versus SD cross sections on ^{27}Al for the cosmologically relevant neutralinos of data sets A, B, C and D without grand unification constraints.

Fig. 10: SI cross section on ^{27}Al versus bino purity Z_{11}^2 for the cosmologically relevant neutralinos of data sets B and D with $\tan\beta < 2$ without grand unification constraints. The points near $Z_{11}^2{=}\cos^2\theta \simeq 0.77$ correspond to neutralinos with a large photino component.

Fig. 11: SI cross section on ^{27}Al versus $\tan\beta$ for the cosmologically relevant neutralinos of data sets B and D without grand unification constraints.

Fig. 12: SD a) and SI b) cross sections on ^{27}Al for the cosmologically relevant neutralinos obtained without grand unification constraints. The data sets A–E are combined.

Fig. 13: SD cross section on ^{27}Al for the cosmologically relevant photinos and binos with purity >0.99. The contribution of the different sets A, B and C is shown. For each set, the cross section of pure photino is ~2 times larger than for pure bino.

Fig. 14: SD cross section on ^{27}Al versus bino purity for the cosmologically relevant neutralinos of data sets B and D without grand unification constraints.

Fig. 15: SD a) and SI b) cross sections on ^{27}Al for the cosmologically relevant neutralinos obtained for the combined data sets A–E with the grand unification constraints.

Fig. 16: Cross section σ_0 (SI+SD) on ^{23}Na and ^{127}I for the cosmologically relevant neutralinos evaluated without grand unification constraints. The sets A–E are combined.

Fig. 17: SD cross section of the cosmologically relevant neutralinos on ^{73}Ge versus bino purity for data sets B and D without grand unification constraints.

Fig. 18: SI cross section versus the SD part on ^{73}Ge for the cosmologically relevant neutralinos of data sets B and D without grand unification constraints.

Fig. 19: SD a) and SI b) cross section on ^{73}Ge versus the neutralino mass for the cosmologically relevant neutralinos of combined data sets A–E without grand unification constraints.

Fig. 20: As in Fig. 19 with grand unification constraints.

Fig. 21: Cross section σ_0 (SI+SD) of the cosmologically relevant neutralinos on ^{117}Sn for the combined sets A–E without grand unification constraints.

Fig. 22: Cross section σ_0 (SI+SD) on ^{27}Al for the cosmologically relevant neutralinos of data sets A–E without grand unification constraints. A rough estimate of the sensitivity of the first phase of the CRESST experiment is also shown for a measurement time of 1 kg-year assuming a flat radioactive background of 1 count/kg/keV/day and a detector energy threshold of 500 eV with a full width half maximum energy resolution ΔE=200 eV.

Fig. 23: Normalized interaction rate, in units of count/kg/day/pbarn, showing form factor and dark matter flux effects. The rate is evaluated with a zero energy threshold. Solid line: rate in a sapphire detector due to interactions on ^{27}Al. Dashed lines: rate in a NaI detector due to interactions on ^{23}Na and on ^{127}I respectively. Dotted lines: rate in 100% enriched ^{73}Ge (upper) and ^{117}Sn (lower) detectors.

Fig. 24: Cross section σ_0 (SI+SD) normalized to the reduced mass squared for different materials and neutralino compositions. Solid triangles: zino-higgsino mixture with 40% zino composition. Open triangles: pure higgsino. Solid circles: pure bino. Open circles: pure photino. The cross sections are calculated for set B with $\tan\beta$=2, M_A=50 GeV and $\alpha = -1.24$. As can be seen in Fig. 4, zino-higgsino mixtures and pure higgsinos are cosmologically relevant above ~20 GeV, while binos and photinos are cosmologically relevant above ~2 GeV (without grand unification constraints).

Tables

Table 1:   Ranges used for the free parameters of the unconstrained MSSM. Calculations with the grand unification constraints are performed using the same ranges but with $M_1/M_2 = 0.5$ and $M_2 \geq 50\,\text{GeV}$.

	min.	max.	
M_2	1	1000	GeV
M_1/M_2	0	1	
μ	-1000	1000	GeV
$\tan\beta$	1	50	

Table 2:   Masses in GeV of the squark M_{sq}, slepton M_{sl}, and CP-odd Higgs M_A used to compute the neutralino relic abundance and the cross section on nuclei for our different trial sets.

set	M_{sq}	M_{sl}	M_A
A	45	45	25-70
B	100	45	25-70
C	150	90	25-70
D	200	200	200
E	1000	1000	1000

Table 3: Axial coupling coefficients $a_{p(n)}$ for pure photino and pure bino evaluated from Eq. (11) using different predictions for the quark spin coefficients ΔQ_q. The coupling coefficients are shown relative to $M_W/M_{sq}=1$.

		NQM	EMC	Global Fit-1	Global Fit-2
	ΔQ_u	0.93	0.78	0.80	0.82
	ΔQ_d	-0.33	-0.5	-0.16	-0.44
	ΔQ_s	0	-0.16	-0.13	-0.11
photino	a_p	-0.175	-0.127	-0.135	-0.141
	a_n	0.020	0.071	0.060	0.054
bino	a_p	-0.119	-0.084	-0.089	-0.094
	a_n	0.008	0.045	0.037	0.033

Table 4:   Axial-vector current coefficients S_A for pure photino and pure bino (purity=1) interactions on the proton-odd nucleus ^{27}Al and on the neutron-odd nucleus ^{73}Ge. The coefficients S_A are evaluated using different nuclear models and the nucleon quark spin content coefficients listed in Tab. 3. The $< S_{p(n)} >$ values used for the Shell Model are unquenched in the case of Al [19] and quenched (Q=0.833) in the case of Ge [20]. The coupling coefficients are evaluated relative to M_W/M_{sq}=1.

	^{27}Al (J=5/2)		^{73}Ge (J=9/2)	
	OGM	Shell Model	OGM	Shell Model
$< S_p >$	0.25	0.3430	0	0.011
$< S_n >$	0	0.0296	0.23	0.468
S_A for pure photino				
NQM	0.0215	0.0396	0.0002	$2.9 \cdot 10^{-8}$
EMC	0.0113	0.0193	0.0026	0.0058
Global Fit-1	0.0127	0.0221	0.0018	0.0036
Global Fit-2	0.0139	0.0245	0.0015	0.0026
S_A for pure bino				
NQM	0.0099	0.0184	$3.3 \cdot 10^{-5}$	$5.5 \cdot 10^{-5}$
EMC	0.0049	0.0084	0.0011	0.0023
Global Fit-1	0.0056	0.0098	0.0007	0.0013
Global Fit-2	0.0062	0.0110	0.0006	0.0009

Table 5: Axial-vector current coefficients S_A for pure photino and pure bino (purity=1) interactions on the proton-odd nuclei ^{23}Na and ^{127}I and on the neutron-odd nucleus ^{117}Sn. The coefficients are evaluated using the OGM for the nuclear structure and the nucleon quark spin coefficients derived from a fit of the present data on polarized lepton-nucleon scattering (Global Fit-2). The coupling coefficients are evaluated relative to $M_W/M_{sq}=1$.

	^{23}Na (J=3/2)	^{127}I (J=5/2)	^{117}Sn (J=1/2)
$< S_p >$	0.156	0.07	0
$< S_n >$	0	0	0.261
S_A for pure photino	0.0065	0.0011	0.0048
S_A for pure bino	0.0029	0.0005	0.0018